Journal of the Korean Physical Society, Vol. 53, No. 6, December 2008, pp. 3640∼3644

Complementarity in a Closed-Loop Aharonov-Bohm Interferometer

Gyong Luck KHYM
Department of Physics, Chonnam National University, Gwangju 500-757 and
Department of Physics, Pohang University of Science and Technology, Pohang 790-784

Kicheon KANG*
Department of Physics, Chonnam National University, Gwangju 500-757

Dong-In CHANG and Hu-Jong LEE
Department of Physics, Pohang University of Science and Technology, Pohang 790-784

Yunchul CHUNG
Department of Physics, Pusan National University, Busan 609-735

(Received 21 August 2007, in final form 9 June 2008)

We study complementarity in a two-terminal closed-loop Aharonov-Bohm interferometer with a quantum dot embedded in an arm of the ring and a nearby charge detector capacitively coupled to the quantum dot. We show that perfect charge detection does not give full information of the electronic path, so it does not completely suppress the interference. We propose that this property can be verified even in the limit of weak charge detection by investigating the dephasing of the first and the second harmonics of the Aharonov-Bohm oscillation. The first harmonic of the AB oscillation is reduced monotonically as a function of the charge detection strength. In contrast, the second harmonic interference remains unaffected due to the fact that charge detection is not able to detect the electronic path responsible for the second harmonic. Our theory confirms that the particle-like behavior of an electron emerges from the path information itself rather than from the disturbance.

PACS numbers: 72.10.Bg, 73.21.La, 03.65.Yz, 03.67.-a
Keywords: Complementarity, Closed-loop Aharonov-Bohm interferometer, Charge detection

I. INTRODUCTION

Complementarity in quantum theory is nicely illustrated in a two-path interferometer with a detector that monitors a particle's path. In a *which-path* interferometer with two possible paths, observation of the interference pattern and obtaining path information cannot be made simultaneously [1–3]. A which-path interferometer was demonstrated by using a pair of correlated photons [4], where the path information of a photon was carried by its entangled twin. The phase coherence of electron transport through a quantum dot (QD) was displayed by the oscillation of the current in an Aharonov-Bohm (AB) interferometer as a function of the magnetic flux [5, 6]. An Electronic which-path interferometer has been realized by using an AB interferometer containing a quantum dot (QD) with a nearby quantum point contact (QPC) [7–9] that is able to detect the charge state of the QD. The which-path detection is made via the QD-charge dependence of the current through the QPC. This results in dephasing of the charge state of the QD and suppression of the AB oscillation. The suppression strength is controlled via the voltage across the QPC. Different setups for controlled dephasing experiment were also demonstrated by using QD-QPC hybrid structures [10,11]. The open-type geometry studied in Refs. 6 and 7 can be regarded as a two-path interferometer because of a low probability of back reflection of an electron to its source electrode. In contrast, the closed-loop geometry with two terminals in Ref. 5 is different from the typical two-path interferometer because of the possible multiple reflections at junctions of the conductor [12].

In this paper, we study the dephasing in a *closed-loop* AB *interferometer* due to a charge state measurement and try to clarify the nature of the 'dephasing induced by detection'. Our results show that perfect charge detection does not completely suppress the interference. In real experiments, it is difficult to realize perfect charge detection in mesoscopic conductors. We propose that our theoretical prediction can be verified even in the limit

*E-mail: kckang@chonnam.ac.kr

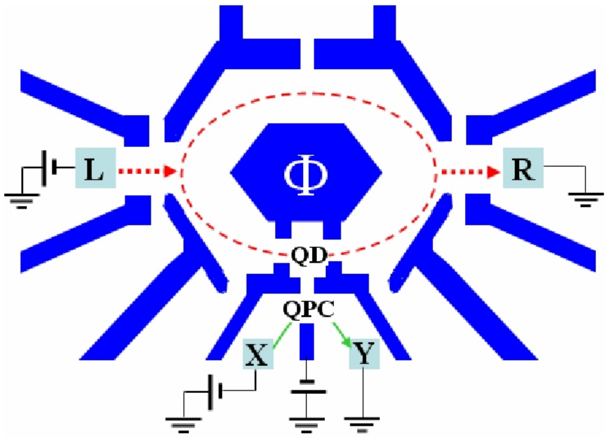

Fig. 1. (Color online) Schematic diagram of the model system: A closed-loop Aharonov-Bohm (AB) interferometer with a quantum dot embedded in its lower arm coupled to a quantum point contact detector.

of weak charge detection by investigating the dephasing of the first and the second harmonics of the Aharonov-Bohm oscillation.

II. CLOSED-LOOP AHARONOV-BOHM INTERFEROMETER UNDER CHARGE MEASUREMENT

Figure 1 shows a schematic diagram of the model system under consideration. A QD is embedded in one arm of a closed-loop AB interferometer with two terminals and is capacitively coupled to a mesoscopic detector. In principle, there are an infinite number of possible trajectories with multiple reflections in this interferometer [13]. In a conventional viewpoint, one assumes that an electron passing through the QD loses its phase coherence upon charge detection. That is, one might expect the strong Coulomb interaction between the two subsystems to give phase randomization of electron transmission through the QD that washes out the AB oscillation. However, it should be noted that there exist indistinguishable paths for an electron passing through the QD, in spite of perfect charge detection (see Figure 2). For those paths drawn in Figure 2(a) and Figure 2(b), a charge detection cannot give any information about the path taken by the electron. In the following, we show that, because of the reason explained above, the interference in the closed-loop interferometer does not vanish, not even in the limit of perfect charge detection.

A closed-loop AB interferometer which has a QD in one of its arms is modeled by using the Hamiltonian $H_{AB} = H_0 + H_T$, where

$$H_0 = \epsilon_d d^\dagger d + \sum_\alpha \sum_k \epsilon_k c^\dagger_{\alpha k} c_{\alpha k} \ , \tag{1a}$$

$$H_T = \sum_k [(W c^\dagger_{Lk} c_{Rk} + V d^\dagger c_{Lk} + V c^\dagger_{Rk} d) + h.c.] \ . \tag{1b}$$

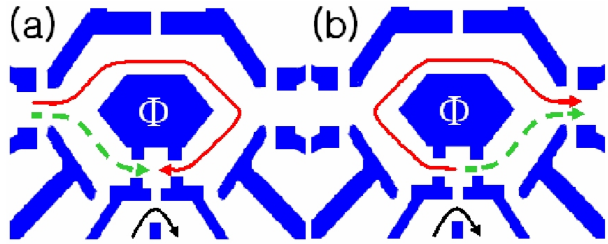

Fig. 2. (Color online) Indistinguishable paths of electron transport upon charge detection (a) from lead L to the QD and (b) from the QD to lead R.

H_0 describes a QD and the two leads L and R. H_T represents hopping of electrons between the subsystems. The operator d ($d^\dagger$) annihilates (creates) an electron with energy ϵ_d in the QD. The operators $c_{\alpha k}$ ($\alpha = L, R$) and $c^\dagger_{\alpha k}$ annihilate and create an electron with energy $\epsilon_{\alpha k}$ at lead α, respectively. The hoping amplitudes V and W are chosen as $V = |V|$ and $W = |W|e^{i\varphi}$, where φ is the phase of an electron state enclosed by the loop of the interferometer.

In the absence of charge detection, we write the retarded Green's function of the closed-loop AB interferometer, $\bar{\mathcal{G}}$, by using a perturbation expansion on the hopping part of the Hamiltonian (H_T):

$$\bar{\mathcal{G}} = \bar{\mathcal{G}}_0 + \bar{\mathcal{G}}_0 H_T \bar{\mathcal{G}}_0 + \bar{\mathcal{G}}_0 H_T \bar{\mathcal{G}}_0 H_T \bar{\mathcal{G}}_0 + \cdots \ , \tag{2}$$

where $\bar{\mathcal{G}}_0^{-1} = \epsilon \mathbf{1} - H_0 + i0^+$. $\bar{\mathcal{G}}$ can be evaluated exactly in the absence of interactions. Nevertheless, the above expansion is crucial for systematic inclusion of the effect of charge detection.

Let us consider the injection of two electrons, one from lead L of the interferometer and the other from lead X of the detector (See Figure 1). Also, our treatment is restricted to the off-resonance limit of the QD, where we can neglect higher order terms (than $|V|^2$) in the expansion. The transmission probability through the QD is very small in the off-resonance limit. Thus, upon a scattering of two electrons, we can write the two-particle state as

$$|\psi\rangle \simeq |\phi_0\rangle_{AB} \otimes |\chi_0\rangle_{PC} + |\phi_1\rangle_{AB} \otimes |\chi_1\rangle_{PC} \ , \tag{3}$$

where $|\phi_0\rangle_{AB}$ denotes all possible paths that do not reach the QD. The state $|\phi_1\rangle_{AB}$ includes all processes that includes the leading (second) order tunneling through the QD. Higher order tunneling processes are neglected in our scheme. Note that $|\phi_0\rangle_{AB}$ and $|\phi_1\rangle_{AB}$ include multiple reflections at the contacts between the leads and the interferometer (See Figure 3). The corresponding detector states, $|\chi_0\rangle_{PC}$ and $|\chi_1\rangle_{PC}$, are given by

$$|\chi_i\rangle_{PC} = \bar{r}_i|X\rangle + \bar{t}_i|Y\rangle \ \ (i = 0, 1), \tag{4}$$

where $\bar{r}_i$ and $\bar{t}_i$ are the i-dependent reflection and transmission amplitudes, respectively. $|X\rangle$ and $|Y\rangle$ are states of the electron at lead X and Y, respectively.

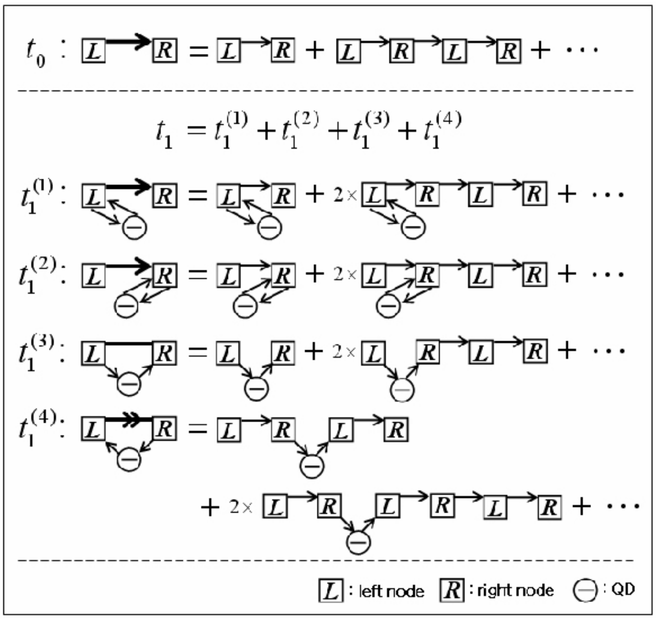

Fig. 3. Diagrams of the possible trajectories from lead L to R. t_0 represents the electron transition through the free arm. t_1 includes all possible diagrams with a single tunneling event through the QD, that can be classified into four different contributions, namely, $t_1^{(1)}$, $t_1^{(2)}$, $t_1^{(3)}$ and $t_1^{(4)}$.

The transmission amplitudes for the states $|\phi_0\rangle_{AB}$ and $|\phi_1\rangle_{AB}$ are just the scalar products between the state $|R\rangle$ (denoting the electron at lead R) and the corresponding electronic states: $t_0 = \langle R|\phi_0\rangle_{AB}$ and $t_1 = \langle R|\phi_1\rangle_{AB}$. By using the relation between Green's function and the transmission amplitude [14,15], we find that

$$t_i = i\hbar v \mathcal{G}_i \quad (i = 0, 1) , \tag{5a}$$

where v is the Fermi velocity. In our case, the Green's function, $\mathcal{G}_i$, (and the transmission coefficient t_i) which correspond to the state $|\phi_i\rangle_{AB}$ is calculated from the expansion of $\bar{\mathcal{G}}$ in Eq. (2). Diagrams for the infinite series of this perturbation expansion are displayed in Figure 3. In terms of this expansion, we get

$$t_0 = \frac{-2ix}{1+x^2} e^{-i\varphi} , \tag{5b}$$

where $x = \pi\rho|W|$, with ρ being the density of states at the Fermi energy. The amplitude t_1 has four different contributions, namely, $t_1 = t_1^{(1)} + t_1^{(2)} + t_1^{(3)} + t_1^{(4)}$, as shown in Figure 3, where

$$\begin{aligned} t_1^{(1)}(\epsilon) &= t_1^{(2)}(\epsilon) = \frac{i\Gamma}{\epsilon_d} t_0(\epsilon) , \\ t_1^{(3)}(\epsilon) &= -\frac{\Gamma}{\epsilon_d} t_0(\epsilon) \frac{e^{i\varphi}}{x} , \\ t_1^{(4)}(\epsilon) &= \frac{\Gamma}{\epsilon_d} t_0(\epsilon) x e^{-i\varphi} , \end{aligned}$$

and $\Gamma = \pi\rho|V|^2/(1+x^2)$ is the effective resonance width of the QD level. The sum of these four amplitudes yields

$$t_1 = \frac{\Gamma}{\epsilon_d} t_0 \left(2i - \frac{e^{i\varphi}}{x} + x e^{-i\varphi} \right) . \tag{5c}$$

For the entangled two-particle state $|\psi\rangle$ of Eq. (3), the probability to find an electron at lead R at zero temperature can be evaluated through the equation

$$T_{AB} = \langle\psi|R\rangle\langle R| \otimes \hat{I}_{PC}|\psi\rangle , \tag{6}$$

where $\hat{I}_{PC}$ is the identity operator that acts only on the QPC detector. It gives

$$T_{AB} = |t_0|^2 + |t_1|^2 + 2\mathrm{Re}[\lambda t_0^* t_1] , \tag{7a}$$

where the parameter λ represents the indistinguishability between the two charge states, $|\phi_0\rangle_{AB}$ and $|\phi_1\rangle_{AB}$, given by

$$\lambda = {}_{PC}\langle\chi_0|\chi_1\rangle_{PC} = \bar{r}_0^* \bar{r}_1 + \bar{t}_0^* \bar{t}_1 . \tag{7b}$$

It should be noted that Eq. (7) is very general and is independent of the type of the detector.

The third term in T_{AB} of Eq. (7a) corresponds to the interference between the paths represented by t_0 and t_1. This term is proportional to $|\lambda|$ as a result of charge detection. In the limit of perfect detection, the two detector states $|\chi_0\rangle_{PC}$ and $|\chi_1\rangle_{PC}$ are orthogonal to each other ($\lambda = 0$) and the third term of Eq. (7a) vanishes. However, it is important to note that the second term $|t_1|^2$ of Eq. (7a) also has a phase dependence, which implies that the interferometer shows an AB oscillation even in the limit of perfect charge detection. This phase coherence is a result of the indistinguishability among the possible paths $t_1^{(0)}$, $t_1^{(2)}$, $t_1^{(3)}$ and $t_1^{(4)}$ in the diagrams of Figure 3. The AB oscillations of the transmission probability for several values of $|\lambda|$ are shown in Figure 4(a). In general, the amplitude of the AB oscillation is reduced as λ decreases. However, the oscillation is not entirely suppressed, not even in the limit of perfect charge detection ($\lambda = 0$), in contrast to that of a 'double-slit' type interferometer. It confirms the above argument that some paths of an electron are still indistinguishable by charge detection in a closed-loop interferometer.

In real experiments, it is technically very difficult to realize perfect charge detection in mesoscopic conductors. Normally, an extra charge in a qubit is detected through partial sensitivity of a nearby mesoscopic conductor [7, 16]. Further, even though one could reach the perfect detection limit, it is another task to confirm our prediction that the AB oscillation of T_{AB} in the closed-loop survives in that limit. In the following, we discuss an easier way to get around this issue by analyzing the first and the second harmonics of the interference, where the perfect detection limit is not necessary.

Figure 5 shows the trajectories that give the first and the second harmonic interferences within our scheme of Eqs. (5)−(7). For the case of the first harmonic interference, the charge detection distinguishes the two paths

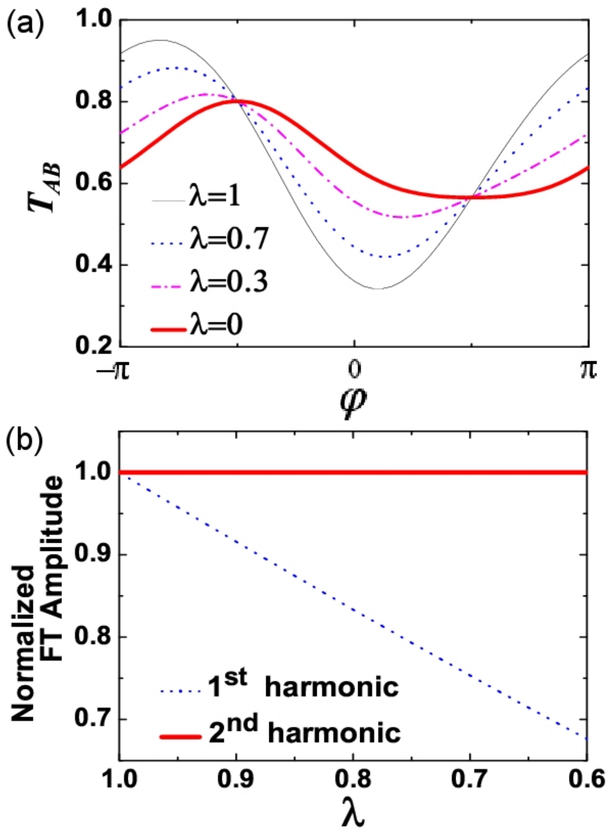

Fig. 4. (Color online) (a) Transmission probability T_{AB} through the closed-loop interferometer as a function of the AB phase. The parameters used here are $x = 0.45$, $V = 0.75|W|$, $\epsilon_d = 1.5|W|$ and $\Gamma/\epsilon_d = 0.4$. (b) Fourier-transformed amplitudes of the first and the second harmonics of T_{AB}. The amplitudes of the two harmonics are normalized to the corresponding values at $\lambda = 1$.

drawn in Figure 5 (a). However, the two paths in Figure 5 (b), which are responsible for the second harmonic interference, are not distinguishable under charge detection because both the paths pass through the quantum dot once. This intriguing property is displayed in the λ-dependence of the Fourier-transformed amplitudes (Figure 4(b)): The first harmonic amplitude decreases monotonically as a function of λ. On the other hand, the second harmonic amplitude remains constant despite charge detection. Recently, this property has been reported experimentally [17].

III. DISCUSSION

It has been widely accepted that the disappearance of interference in a two-path interferometer under path detection could be explained in terms of the inevitable momentum kick enforced by the detection process [2]. With our setup, one might assume a picture that the local Coulomb interaction between the electrons in the QD

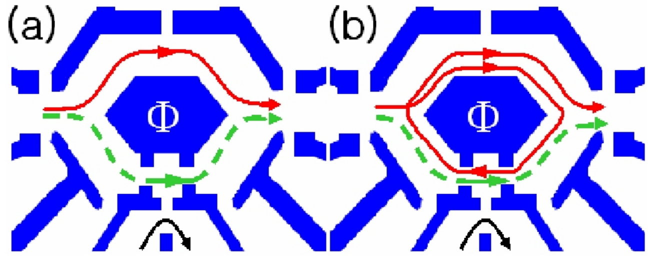

Fig. 5. (Color online) Electronic paths that give (a) the first and (b) the second harmonic interferences, respectively.

and in the detector disturbs the motion of an electron in the QD and results in a randomization of the electron's phase that washes out the interference. According to this picture, however, any electron in the closed-loop interferometer would loose its phase coherence whenever it passes through the QD and the AB oscillation of the closed-loop interferometer would vanish in the strong detection limit. Then, the closed-loop interferometer would not show any essential difference from the double-slit interferometer, in spite of the nontrivial electronic paths. Our results (Eq. (7) and Figure 4) clearly indicate that this is not the case.

IV. CONCLUSION

In conclusion, we have investigated how to study complementarity in a mesoscopic Aharonov-Bohm interferometer with a closed-loop geometry. While the conventional picture of 'two-path' interference is not valid, the closed-loop interferometer provides new insight into understanding the nature of the which-path detection and dephasing. We have found that charge detection reduces, but does not completely suppress, the interference, not even in the limit of perfect detection, which originates from the fact that charge detection does not extract full information on the electron's path. We have discussed how this property can be tested in a weak detection limit by investigating the Fourier-transformed amplitudes of the first and the second harmonic interferences. The first harmonic amplitude is reduced as a function of the charge detection strength, but the second harmonics remains unaffected in the limit of small transmission probability through the quantum dot. Our study confirms that the information itself is more fundamental in understanding quantum-mechanical complementarity.

ACKNOWLEDGMENTS

This work was supported by the Korea Research Foundation (KRF-2005-070-C00055) and by the 'Cooperative Research Program' of the Korea Research Institute of Standards and Science.

REFERENCES

[1] N. Bohr, in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton Univ. Press, New Jersey, 1983), p.9.

[2] R. Feynman, R. Leighton and M. Sands, *The Feynman Lectures on Physics* Vol. III (Addison Wesley, Reading, 1965).

[3] A. Stern, Y. Aharonov and Y. Imry, Phys. Rev. A **41**, 3436 (1990).

[4] X. Y. Zou, L. J. Wang and L. Mandel, Phys. Rev. Lett. **67**, 318 (1991).

[5] A. Yacoby, M. Heiblum, D. Mahalu and H. Shtrikman, Phys. Rev. Lett. **74**, 4047 (1995).

[6] R. Schuster, E. Buks, M. Heiblum, D. Mahadu, V. Umansky and H. Shtrikman, Nature **385**, 417 (1997).

[7] E. Buks, R. Schuster, M. Heiblum, D. Mahalu and V. Umansky, Nature **391**, 871 (1998).

[8] Y. Lee, G. L. Khym and K. Kang, J. Korean Phys. Soc. **51**, 2004 (2007).

[9] D. I. Chang *et al.*, J. Korean Phys. Soc. **51**, 1395 (2007).

[10] D. Sprinzak, E. Buks, M. Heiblum and H. Shtrikman, Phys. Rev. Lett. **84**, 5820 (2000).

[11] M. Avinun-Kalish, M. Heiblum, A. Silva, D. Mahalu and V. Umansky, Phys. Rev. Lett. **92**, 156801 (2004).

[12] G. L. Khym and K. Kang, Phys. Rev. B **74** 153309 (2006).

[13] A. L. Yeyati and M. Büttiker, Phys. Rev. B **52**, R14 360 (1995).

[14] D. S. Fisher and P. A. Lee, Phys. Rev. B **23** 6851 (1981).

[15] T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).

[16] M. Field *et al.*, Phys. Rev. Lett. **70**, 1311 (1993).

[17] D.-I. Chang, G. L. Khym, K. Kang, Y. Chung, H.-J. Lee, M. Seo, M. Heiblum, D. Mahalu and V. Umansky, Nature Physics **4**, 205 (2008).