Journal of the Korean Physical Society, Vol. 63, No. 9, November 2013, pp. 1768∼1771

Magnetic Field Dependence of a Charge-frustrated State in a Triangular Triple Quantum Dot

M. Seo and Y. Chung*
Department of Physics, Pusan National University, Busan 609-735, Korea

(Received 25 July 2013)

We studied the magnetic field dependence of a charge-frustrated state formed in a triangular triple quantum dot. Stability diagrams at various magnetic fields were measured by using two-terminal and three-terminal conductance measurement schemes. We found that the frustrated state broke down at an external magnetic field of around 0.1 T. This result is due to the confinement energy shifts in quantum dots under external magnetic fields. A similar breakdown of the frustrated state was observed when the confinement energy of a quantum dot was intentionally shifted by the plunger gate of the dot, which confirm the reason for the breakdown of the frustrated state under on applied magnetic field. Our measured stability diagrams differed depending on the measurement schemes, which could not be explained by the capacitive interaction model based on an independent particle picture. We believe that the discrepancy is related to the closed electron and hole trajectories inside a triple quantum dot.

PACS numbers: 73.63.Kv, 75.10.Jm, 85.35.Be
Keywords: Triple quantum dots, Charge-frustrated state, Magenetic field dependence
DOI: 10.3938/jkps.63.1768

I. INTRODUCTION

Geometric frustrations are usually found in a condensed matter system where the interactions between different neighboring lattice sites conflict due to the geometrical configuration (*e.g.*, a triangular lattice) [1]. The earliest prediction of such a phenomenon was made by Linus Pauling to explain the residual entropy observed in water ice at a temperature of absolute zero [2]. Geometric frustration has revealed many exotic states such as spin ice [3, 4], spin liquid [5, 6], and the spin ice magnetic monopole [7]. However, these phenomena have been studied only in ensemble systems.

Many interesting phenomena found in ensemble systems have been observed in more controllable systems such as artificial atoms and molecules based on quantum dots [8]. Studies have revealed the Kondo effect [9, 10] energy structure in an artificial atom [11] and molecular bonding states in double quantum dots [12–14]. Because experimental parameters (*e.g.*, inter-dot tunneling and the energy level) can be easily tuned *in situ*, a quantum dot system is an ideal way to study quantum and many-body physics.

Recently, we reported the realization of an isolated single charge-frustrated state in a triangular triple quantum dot (TQD) [15]. The state is found to be degenerate with six-fold charge configurations, which is the signature of frustration. Also, omnidirectional resonant tunneling through the charge-frustrated state has been observed and explained by using the capacitive interaction model [16]. All the phenomena observed can be well explained by using capacitance model, except for unexpected tunneling through forbidden energy states around the frustrated state.

In this work, we studied the magnetic field dependence of the charge-frustrated state. The six-fold charge configurations of a frustrated state allow an electron or a hole to rotate and form closed trajectories inside a TQD. Hence, studying the magnetic field dependence of a frustrated state due to its closed electron and hole trajectories would be interesting. To study the effect of the closed trajectories we measured the stability diagrams for various magnetic fields by using two-terminal and three-terminal conductance measurement schemes. Also the stability diagrams when the TQD was slightly detuned form the frustration condition were measured to study the effect of detuning on the confinement energies.

II. EXPERIMENTS AND RESULTS

A conventional uniformly-doped GaAs/$Al_{0.34}Ga_{0.66}As$ heterostructure grown by molecular beam epitaxy was used to fabricate the triple quantum dot (TQD) shown in Fig. 1(a). A two-dimensional electron gas (2DEG) was buried 77 nm below the wafer's surface. The carrier

*E-mail: ycchung@pusan.ac.kr; Fax: +82-51-513-7664

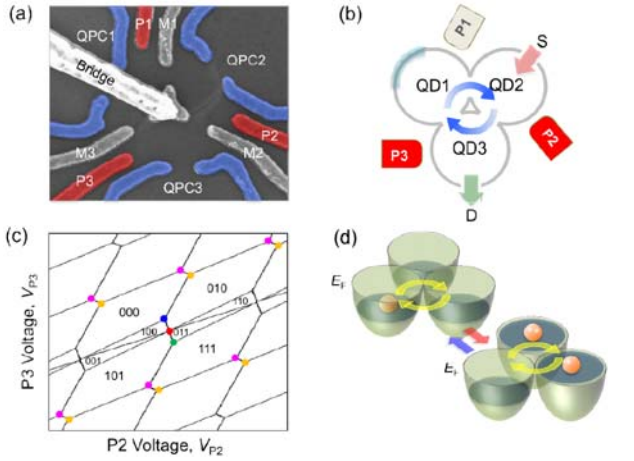

Fig. 1. (Color online) (a) SEM picture of a triangular triple quantum dot. The middle gates (M1, M2, M3) control the coupling between quantum dots. The plunger gates (P1, P2, P3) change the energy states in corresponding quantum dots. QPCs control the coupling between the quantum dot and the reservoir. (b) Schematic diagram of the measurement. The bias voltage was applied to the source S, and the current through the device was measured at drain D. (c) Stability diagram of a TQD in a charge-frustrated state. The occupation numbers in the stability diagram are labeled for clarity (*e.g.*, (0, 0, 0)) by subtracting arbitrary constant numbers from the actual electron occupation numbers (which are positive) in the TQD. (d) Illustration showing the possible charge configurations and the chemical potentials of a TQD in a charge-frustrated state.

density and the mobility were 1.9×10^{11} cm^{-2} and 1.1×10^6 cm^2/Vs, respectively, at 4.2 K. The TQD was defined by using electron beam lithography, and 15/30 nm-thick Ti/Au was used for the gates. All of the measurements were performed in a dilution refrigerator at a base temperature of 50 mK. To determine the stability diagram of the TQD, we measured the conductance $G = dI_{sd}/dV_{sd}$ by applying an excitation voltage of 10 μVrms between the source and drain, and we measured the current by using a device with the standard lock-in technique at 970 Hz. A homemade wide-band low-noise current amplifier, which is capable of taking around 20 conductance data points per second [17], was used to measure the current.

Figure 1(c) shows the calculated stability diagram of the TQD in a charge-frustrated state plotted as a function of the plunger P2 and P3 voltages, as previously reported [15]. The stability diagram, calculated using the capacitive interaction model [16], shows the coexistence of six different charge configurations–(100), (010), (001) and (011), (101), (011)–on the red point, which is the charge-frustrated state. A sequential tunneling process similar to that of a single quantum dot was reported to exist and to allow omnidirectional transport among the three reservoirs without energy cost (*i.e.*, transport between any two of the three reservoirs).

The blue and the green points in Fig. 1(c) are the electron and the hole triple points of quantum dot 1 (QD1) and quantum dot 2 (QD2), which also allow sequential tunneling. The pink and the orange points are the ordinary electron and hole triple points of QD2 and quantum dot 3 (QD3). Except for these special points, the tunneling through a TQD is forbidden, in principle, according to the capacitive interaction model. However, unexpected strong tunneling through a TQD was reported in the regions marked as (100), (001) and (110), (011), including the boundaries. Such tunneling was partly explained by the elastic co-tunneling from QD1 to QD3 via the (100) and (001) states [15]. To avoid such unwanted tunneling, we isolated QD1 from the electron reservoir by closing quantum point contact 1 (QPC1) as shown in Fig. 1(b).

The six-fold charge configurations and the corresponding chemical potential of the TQD are illustrated in Fig. 1(d). The electrostatic energies of the TQD are the same for one and two electrons in the TQD and do not depend on the specific location of the electrons in the TQD. This means that the (100), (010), (001) and (011), (101), (011) charge configurations have the same electrostatic energy, which allows six-fold charge configurations at the frustrated state. However, the chemical potential of the TQD is different depending on the number of electrons in the dot. Figure 1(d) shows the chemical potential of each quantum dot for one and two electrons in the TQD. When the TQD is occupied by one electron only, the chemical potentials of all three dots lie below the Fermi energy of the reservoir [upper left of Fig. 1(d)]. For two electrons in the TQD, the chemical potentials of the dots with electrons rise to match the Fermi energy of the reservoir [lower right of Fig. 1(d)]. Hence, when the TQD is occupied by only one electron, another electron can tunnel into the TQD without energy cost because the electrostatic energy of the TQD is the same for one or two electrons in the dot. Once the TQD has two electrons, an electron can tunnel out because the chemical potentials of the dots with electrons are aligned to the Fermi energy of the reservoir. Note that the electrons in the TQD can freely rotate inside the TQD until one electron tunnels out from the dot. Due to the closed, encircling electron trajectory circular traveled by the electron inside the TQD, the energy states of the TQD can change when a magnetic field is applied perpendicular to the TQD.

The stability diagrams measured for various magnetic fields are shown in Fig. 2. As shown in the figure, the electron and the hole triple points of QD1 and QD2 rotate clockwise around the frustrated point. In addition, all of the triple points of QD2 and QD3 rotate around their centers as the magnetic field increases. The hole triple point of QD1 and QD2 [the conducting point just below the frustrated point, shown by the green point in Fig. 1(c)] becomes less conducting as the magnetic field increases and almost vanishes at 0.11 T. At 0.12 T, the stability diagram looks almost identical to that of a double quantum dot except for a sharp discontinuity on the

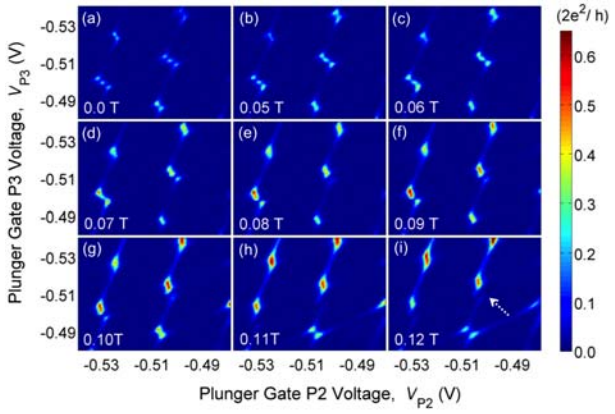

Fig. 2. (Color online) Stability diagrams measured using two-terminal measurements at various magnetic fields. The magnetic field was applied perpendicular to a TQD. The plunger gate voltages of quantum dots QD2 and QD3 were varied while the plunger gate voltage of QD1 was fixed at –0.308 V.

co-tunneling line [shown by the white dotted arrow in Fig. 2(i)], which will be discussed later in this paper.

A possible explanation for the rotation of the triple points is the shifts of the confinement energies in the quantum dots due to the magnetic field. The energy states of a quantum dot are well known to change under an external magnetic field. The energy states can be calculated by treating the potential of a single quantum dot as a 2-D harmonic oscillator, which is known as the Fock-Darwin state [18,19]. The energy states at a specific magnetic field depend on the size of the 2-D potential well. Even though the quantum dots forming the TQD look similar, the actual size of the individual dots could be different due to different gate voltages, the impurity potential from the modulation-doping layer. Hence, the confinement energy shifts of each quantum dot of the TQD could be different from each other. Because the triple and the frustrated points appear when the energy states of the relevant quantum dots are precisely aligned, even a small energy misalignment between quantum dots can change the positions of the triple and the frustrated points in the stability diagram.

Figure 3 shows the stability diagrams measured by changing the energy states inside QD1 to test if the energy misalignment causes the rotations shown in Fig. 2. Figure 3(a) shows the stability diagram for the case in which the TQD is in a frustrated state. The plunger gate voltage of QD1 was varied from the frustration voltage condition (V_{P1} = –0.308 V). Figure 3(e) shows the stability diagram when QD1 is totally detuned from the frustration condition. As shown in the figures, no sign of rotation of the triple points is observed under an magnetic field.

As the plunger gate voltage of QD1 was detuned from the frustration condition, the hole triple points of QD1 and QD2 became less conducting and almost vanished at

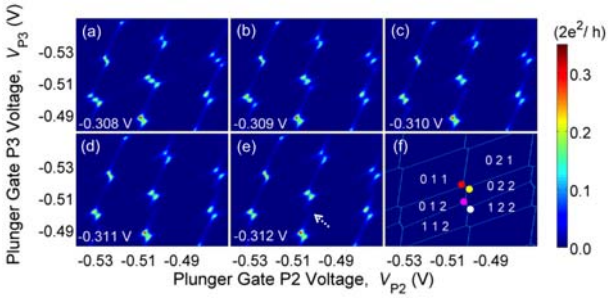

Fig. 3. (Color online) (a)-(e) Stability diagrams measured at various QD1 plunger gate voltages (V_{P1} = –0.308 V ∼ –0.312 V). (f) Stability diagram calculated when the TQD is detuned from the frustration condition. The red and the yellow (pink and white) points are the electron and the hole triple points of QD1 and QD3 (QD1 and QD2), respectively.

–0.310 V. Also, a sharp discontinuity on the co-tunneling line (shown by the white dotted arrow in Fig. 3(e)), which was also seen under an external magnetic field, was observed. Figure 3(f) shows the calculated stability diagram when the plunger gate voltage of QD1 was detuned from the frustration condition. The result is similar to the experimental result measured for V_{P1} = –0.312 V. Four triple points are seen in Fig. 3(f). The top two triple points are the electron (red point) and the hole (yellow point) triple points of QD2 and QD3 while the bottom two are the electron (pink point) and the hole (white point) triple points of QD1 and QD2, as represented by the charge configurations denoted in the figure. Because the conductance was measured through QD2 and QD3 with QD1 being disconnected from the electron reservoir, we expected strong (weak) conductance for the QD2 and the QD3 (QD1) triple points. Hence, the discontinuity on the co-tunneling line is simply due to the weak conductance through the triple points of QD1 and QD2. The existence of the same discontinuity for external magnetic fields suggests that the breakdown of the frustrated state under external magnetic fields is due to a confinement energy shift in the quantum dots. However, the rotations of triple points under magnetic fields cannot be explained by the confinement energy shift in the QDs.

As shown in Fig. 1(d), an electron and a hole can freely rotate in the TQD at the frustrated state. Let us assume that the rotations of triple points under a magnetic field are somehow related to the closed electron and hole trajectories inside the TQD. This assumption can be easily checked by measuring the stability diagram when QPC1 is open to couple QD1 to the electron reservoir because electrons can partly leak through QPC1 to the reservoir to destroy the closed trajectories. The stability diagrams were measured for various magnetic fields as shown in Fig. 4. No visible rotations of the triple points were observed. This suggests that the rotations of the triple points are related to the existence of closed electron and hole trajectories inside the TQD. Hence, we suspect

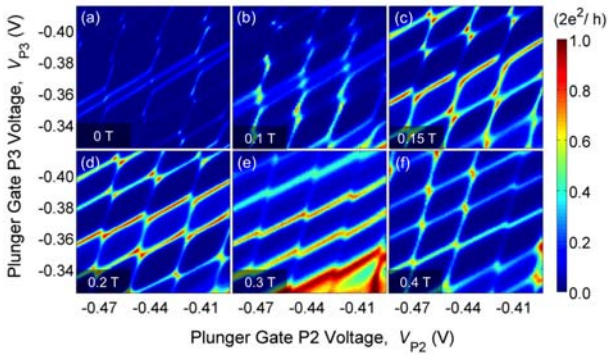

Fig. 4. (Color online) Stability diagrams measured using three-terminal measurements at various magnetic fields. QPC1 was opened to couple QD1 to the electron reservoir.

that the Aharonov-Bohm interference of the electron or the hole, moving in closed circular trajectories inside the TQD, could be responsible for the observed rotations.

III. SUMMARY AND CONCLUSION

We examined the magnetic field dependence of a charge-frustrated state formed in a triangular triple quantum dot. The stability diagrams for various magnetic fields were measured by using two-terminal and three-terminal conductance measurement schemes. We found that the stability diagrams differed depending on the measurement scheme. In both cases, the charge frustration broke down at a magnetic field of about 0.1 T, which could be ascribed to the shift of confinement energies in the quantum dots due to the external magnetic field. However, unexpected rotations of triple points were observed in the stability diagram determined by using the two-terminal measurement. This result could not be explained using the capacitive interaction model based on the independent particle picture. We believe that the discrepancy is related to the closed electron and hole trajectories inside a triple quantum dot.

ACKNOWLEDGMENTS

We thank M. Heiblum for discussion and experimental support (access to his facilities). Also, we thank D. Mahalu and V. Umansky for their experimental support. This work was supported for two years by a Pusan National University research grant.

REFERENCES

[1] N. P. Ong and R. J. Cava, Science **305**, 52 (2004).
[2] L. Pauling, J. Am. Chem. Soc. **57**, 2680 (1935).
[3] S. T. Bramwell and M. J. P. Gingras, Science **294**, 1495 (2001).
[4] A. P. Ramirez, A. Hayashi, R. J. Cava, R. B. Siddharthan and S. Shastry, Nature **399**, 333 (1999).
[5] L. Balents, Nature **464**, 199 (2010).
[6] F. L. Pratt, P. J. Baker, S. J. Blundell, T. Lancaster, S. Ohira-Kawamura, C. Baines, Y. Shimizu, K. Kanoda, I. Watanabe and G. Saito, Nature **471**, 612 (2011).
[7] S. T. Bramwell, S. R. Giblin, S. Calder, R. Aldus, D. Prabhakaran and T. Fennell, Nature **461**, 956 (2009).
[8] U. Meirav, M. A. Kastner and S. J. Wind, Phys. Rev. Lett. **65**, 771 (1990).
[9] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav and M. Kastner, Nature **391**, 156 (1998).
[10] W. G. Van der Wiel, S. De Franceschi, T. Fujisawa, J. M. Elzerman, S. Tarucha and L. P. Kouwenhoven, Science **289**, 2105 (2000).
[11] S. Tarucha, D. G. Austing, T. Honda, R. J. van der Hage and L. P. Kouwenhoven, Phys. Rev. Lett. **77**, 3613 (1996).
[12] C. Livermore, C. H. Crouch, R. M. Westervelt, K. L. Campman and A. C. Gossard, Science **274**, 1332 (1996).
[13] T. H. Oosterkamp, T. Fujisawa, W. Van Der Wiel, K. Ishibashi, R. Hijman, S. Tarucha and L. P. Kouwenhoven, Nature **395**, 873 (1998).
[14] T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha and L. P. Kouwenhoven, Science **282**, 932 (1998)
[15] M. Seo, H. K. Choi, S.-Y. Lee, N. Kim, Y. Chung, H.-S. Sim, V. Umansky and D. Mahalu, Phys. Rev. Lett. **110**, 046803 (2013).
[16] D. Schröer, A. D. Greentree, L. Gaudreau, K. Eberl, L. C. L. Hollenberg, J. P. Kotthaus and S. Ludwig, Phys. Rev. B **76**, 075306 (2007).
[17] A. V. Kretinin and Y. Chung, Rev. Sci. Inst. **83**, 084704 (2012).
[18] V. Fock, Z. Phys. **47**, 446 (1928); C. G. Darwin, Proc. Camb. Phil. Soc. **27**, 86 (1930).
[19] L. P. Kouwenhoven, D. Austing and S. Tarucha, Rep. Prog. Phys. **64**, 701 (2001).